E-LEARNING NOTES

In

Thermodynamics

Topics

Thermodynamics Cycles, Heat Engines and Refrigerator

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1.1 Introduction

In my rarefied, theoretical, academic and unpractical mind, a heat engine consists of a working substance obeying some idealized equation of state such as that for an ideal gas, held inside a cylinder by a piston, and undergoing, in a closed cycle, a series of highly idealized processes, such as reversible adiabatic expansions or isothermal compressions. At various stages of the cycle, the system may be gaining heat from or losing heat to its surroundings; or we may be doing work on the system by compressing it, or the system may be expanding and doing external work.

The *efficiency* η of a heat engine is defined as

$$\eta = \frac{\text{net external work done } \textbf{by} \text{ the engine during a cycle}}{\text{heat supplied } \textbf{to} \text{ the engine during a cycle}}.$$

By "net" external work, I mean the work done **by** the engine during that part of the cycle when it is doing work *minus* the work done **on** the engine during that part of the cycle when work is being done on it. Notice that the word "net" does not appear in the denominator, which refers only to the heat supplied **to** the engine during that part of the cycle when it is gaining heat.

During the compression part of the cycle, the system gives out heat, and only the difference "heat in *minus* heat out" is available to do the external work. Thus efficiency can also be calculated from

$$\eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}},$$

although the *definition* of efficiency remains as equation

No heat engine is 100% efficient, and we need to ask what is the most efficient heat engine possible, what are the factors that limit its efficiency, and what is the greatest possible efficiency? Obviously things like friction in the moving parts of the engine limit the efficiency, but in my academic mind the engine is built with frictionless bearings and all processes in the cycle of compressions and expansions are reversible.

During a cycle, a heat engine moves in a clockwise closed path in the *PV* plane, and, if the processes are reversible, the area enclosed by this clockwise path is the net external work done **by** the system. It also moves in a clockwise closed path in the *TS* plane, and, if the processes are reversible, the area enclosed by this clockwise path is the net heat supplied **to** the system. The two are equal, and when the system returns to its original state, there is no change in the internal energy. That is, internal energy is a function of state.

Depending upon the nature of the various processes during the cycle, the cycle may carry various names, such as the Carnot, Stirling, Otto, Diesel or Rankine cycles. Of these, the most important from the theoretical point of view is the Carnot cycle. I do not know whether anyone has ever built a Carnot heat engine. I do know, however, that no one has ever built an engine working between a hot heat source and a cold heat sink that is more efficient than a Carnot engine; for, for a given temperature difference between source and sink, the Carnot engine is the most efficient conceivable. There is another important thing about the Carnot cycle. In Chapter 3, we struggled to understand that most difficult of all the thermodynamic concepts, namely *temperature*, and we wondered if we could define an *absolute* temperature scale that was independent of the properties of any particular substance. Consideration of the Carnot cycle enables us to do just that.

Of real heat engines I know very little. I know that one pedal of my car makes the car go faster and the other makes it go slower – but what is under the hood or bonnet is beyond my ken. Real heat engines may resemble some of the theoretical engines of academia to a greater or lesser extent. Thus a motor car engine may resemble an Otto cycle, or a steam engine may resemble a Rankine cycle, or a real Diesel engine may resemble the theoretical Diesel cycle. Engineering students may wonder whether they need bother with learning about "theoretical" engines that bear little resemblance to the metal and fuel that they have to work with on a practical basis. I cannot answer that, but there is just one thing I *do* know about real engines, and that is that they are subject to and follow all the fundamental laws of thermodynamics that theoretical engines have to follow; and I suspect that the engineer who designed the engine in my car had a pretty thorough knowledge of the fundamental principles of thermodynamics.

1.2 Carnot Cycle

I referred above to one of the uses of the theoretical concept known as the *Carnot cycle*, namely that it enables us to define an absolute temperature scale.

As a temporary measure I am going to use the symbol θ to represent the temperature measured on the ideal gas scale. I shall then define an *absolute* temperature scale, *T*, and show that it is identical with the ideal gas temperature scale.

To start with, I shall suppose that the working substance in our Carnot engine is an ideal gas. We shall refer to figure , in which *ab* and *cd* are isotherms at temperatures θ_2 and θ_1 respectively ($\theta_2 > \theta_1$), and *bc* and *da* are adiabats. Starting at the point $a(P_1, V_1)$, a quantity of heat Q_2 is supplied **to** the gas as it expands isothermally from *a* to $b(P_2, V_2)$

at temperature θ_2 on the ideal gas scale. During this phase, the cylinder is supposed to be uninsulated and placed in a hot bath at temperature θ_2. As it expands isothermally it does external work. Since the working substance is an ideal gas, the internal energy at constant temperature is independent of volume (there is no internal work against van der Waals forces to be done) so the heat supplied **to** the gas is equal to the external work that it does. That is, per mole,

$$Q_2 = R\theta_2 \ln(V_2/V_1).$$

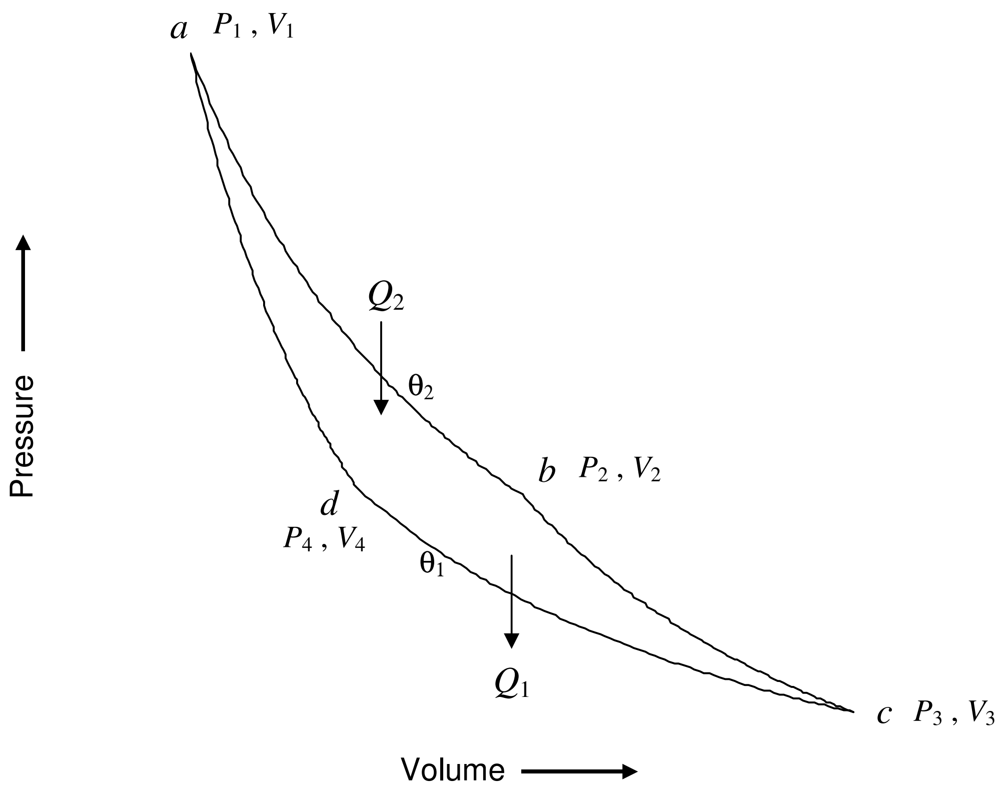


Figure 1

After the gas has reached *b* the cylinder is insulated and the gas expands adiabatically and reversibly to *c* (P_3 , V_3).

It is then placed in a cold bath at temperature θ_1, uninsulated, and compressed isothermally to *d* (P_4 , V_4). During this stage it gives out a quantity of heat Q_1:

$$Q_1 = R\theta_1 \ln(V_3/V_4).$$

Finally it is insulated again and compressed adiabatically and reversibly to its original state *a*.

For these four stages we have the equations

$$P_1V_1 = P_2V_2 ,$$

$$P_2V_2^{\gamma} = P_3V_3^{\gamma} ,$$

$$P_3V_3 = P_4V_4 ,$$

$$P_1V_1^{\gamma} = P_4V_4^{\gamma} .$$

From these, we readily see that

$$V_2 / V_1 = V_3 / V_4 ,$$

and therefore $$Q_2 / Q_1 = \theta_2 / \theta_1 .$$

The net heat received is $Q_2 - Q_1$, and this is the heat available for doing external work. A quantity of heat must be supplied at the beginning of each cycle, and so the efficiency of the cycle is

$$\eta = \frac{Q_2 - Q_1}{Q_2} = \frac{\theta_2 - \theta_1}{\theta_2} .$$

Thus the *efficiency* of the Carnot engine is the *fractional temperature difference between source and sink.*

We have specified in the above that the working substance is an ideal gas, the temperatures of source and sink being θ_1 and θ_2 on the ideal gas scale. Let us now not specify what the working substance is, but let us set up a system of 100 Carnot engines working in tandem, with the sink of one being the source for the next. We'll have the sink for the coldest engine in a bucket of melting ice (0 $^{\circ}$C) and the source for the hottest engine in a bucket of boiling water (100 $^{\circ}$C). They will be working between isothermals and adiabats on an absolute thermodynamic scale, *T*, defined such that the net work done by each engine (i.e. the area of each *PV* loop) per cycle is the same for each of the engines. This will define the temperature on an absolute scale. It would take me a while to use the computer to do a decent drawing of 100 isotherms and 2 adiabats, so I'm going to try to make do with a hand-drawn sketch (figure) of just five isotherms, two adiabats and four linked Carnot cycles to illustrate what I am trying to describe.

We suppose that the efficiency of such a Carnot engine depends solely on the temperature of source and sink:

$$Q_1 / Q_2 = f(T_1 , T_2).$$

We are making no assumption about the form of this function, which is completely arbitrary. We are free to define it in any manner that is useful to us in our attempt to define an absolute temperature scale.

Let us consider two adjacent engines, one working between temperatures T_1 and T_2, and the other working between temperatures T_2 and T_3. We have:

$$Q_1 / Q_2 = f(T_1, T_2),$$

$$Q_2 / Q_3 = f(T_2, T_3),$$

and for the pair as a whole considered as a single engine,

$$Q_1 / Q_3 = f(T_1, T_3).$$

From these we find that

$$f(T_1, T_2) = \frac{f(T_1, T_3)}{f(T_2, T_3)}.$$

This can be only if T_3 cancels from the right hand side, so that

$$f(T_1, T_2) = \frac{\phi(T_1)}{\phi(T_2)}.$$

That is,
$$\frac{Q_1}{Q_2} = \frac{\phi(T_1)}{\phi(T_2)}.$$

And since ϕ is a completely arbitrary function that we can choose at our pleasure to define an absolute scale, we choose

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}.$$

And, with this choice, the absolute thermodynamic temperature scale is identical with the ideal gas temperature scale. Equation 11.2.17 also implies that entropy in = entropy out. Entropy is conserved around the complete cycle. Entropy is a function of state.

In Sections 11.3 to 11.5 I give examples of some other cycles. These are largely for reference, and readers who wish to continue without interruption with the theoretical development of the subject can safely skip these and move on to Sections 11.7 and 11.8.

1.3 Stirling Cycle

This takes place between two isotherms and two isochors. Note that, provided the working substance is an ideal gas, there is no change in the internal energy along the isotherms, and that the work done by or on the gas is equal to the heat gained by or lost from it. No work is done along the isochors. I show the cycle in the *PV* plane in figure XI.3, and an imaginary schematic engine in figure

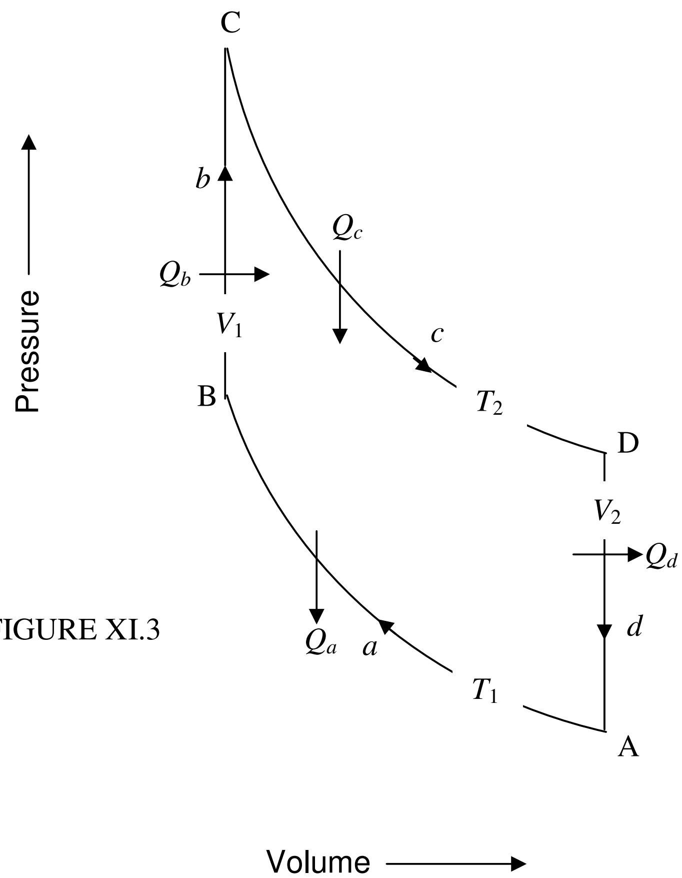


FIGURE XI.3

$$Q_a = RT_1 \ln(V_2/V_1)$$

$$Q_b = C_V(T_2 - T_1)$$

$$Q_c = RT_2 \ln(V_2/V_1)$$

$$Q_d = C_V(T_2 - T_1)$$

Figure 2

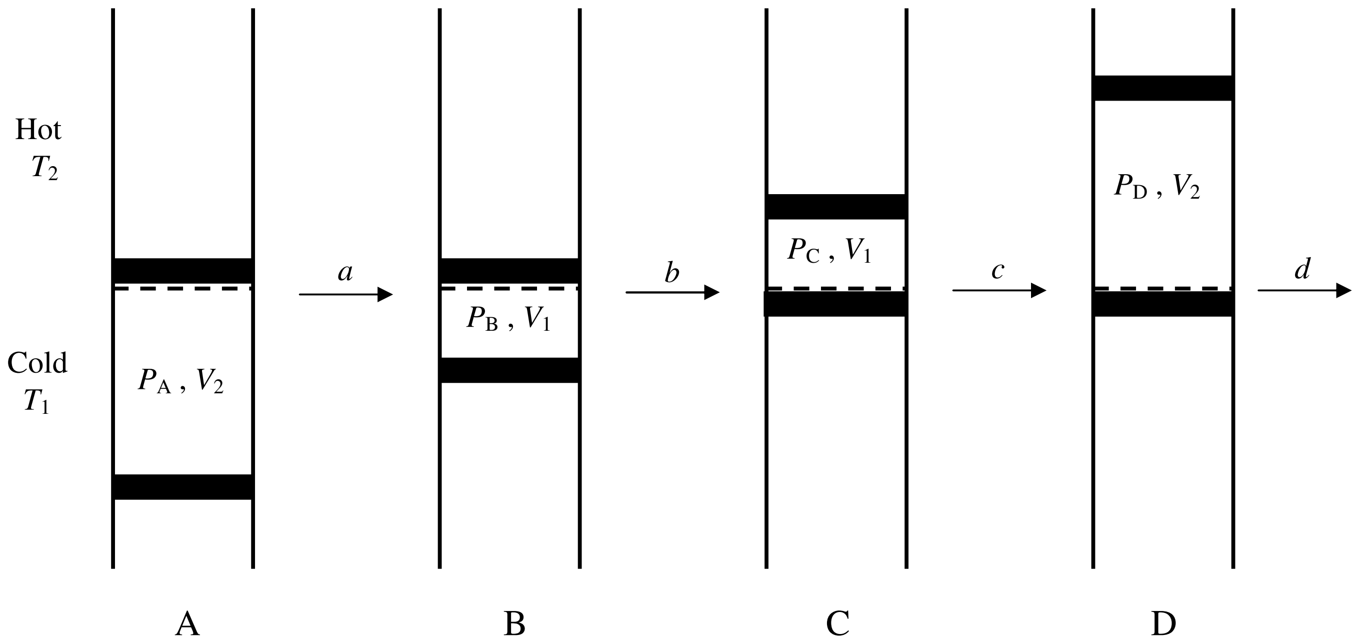


Figure 3

The gas is supposed to be held in a cylinder between two pistons. The cylinder is divided into two sections by a porous partition. One section is kept at a hot temperature T_2 and the other is kept at a cold temperature T_1.

In stage *a*, the cold gas is compressed isothermally. The work done **on** a mole of the gas is $RT_1 \ln(V_2 / V_1)$; this is converted into heat, Q_a, which is lost from the gas to the cold reservoir.

In stage *b*, the gas, held at constant volume, is transferred to the hot reservoir. No work is done on or by the gas, but a quantity of heat $Q_b = C_V (T_2 - T_1)$ per mole is supplied **to** the gas.

In stage *c*, the hot gas is expanded isothermally to its original volume. The work done **by** a mole of the gas is $RT_2 \ln(V_2 / V_1)$; in order to prevent the gas from cooling down, it has to absorb an equal amount of heat, Q_c **from** the hot reservoir. Note that $Q_c > Q_a$.

In stage *d*, the gas, held at constant volume, is transferred back to the cold reservoir. No work is done on or by the gas, but the gas **loses** a quantity of heat $Q_d = C_V(T_2 - T_1)$ to the cold reservoir. Note that $Q_d = Q_b$.

Exercise: Show that the efficiency is

$$\eta = \frac{R(T_2 - T_1)\ln(V_2/V_1)}{C_V(T_2 - T_1) + RT_2\ln(V_2/V_1)}.$$

If the gas is an ideal diatomic gas (to which air is an approximation), then $C_V = \frac{5}{2}R$, and then

$$\eta = \frac{(T_2 - T_1)\ln(V_2/V_1)}{2.5(T_2 - T_1) + T_2\ln(V_2/V_1)}.$$

If helium were used as an ideal gas, the efficiency would be greater, because for helium, $C_V = \frac{3}{2}R.$

1.4 Otto Cycle

The Otto cycle (to which the engine under the hood of your car bears some slight resemblance) works between two isochors and two adiabats (figure XI.5).

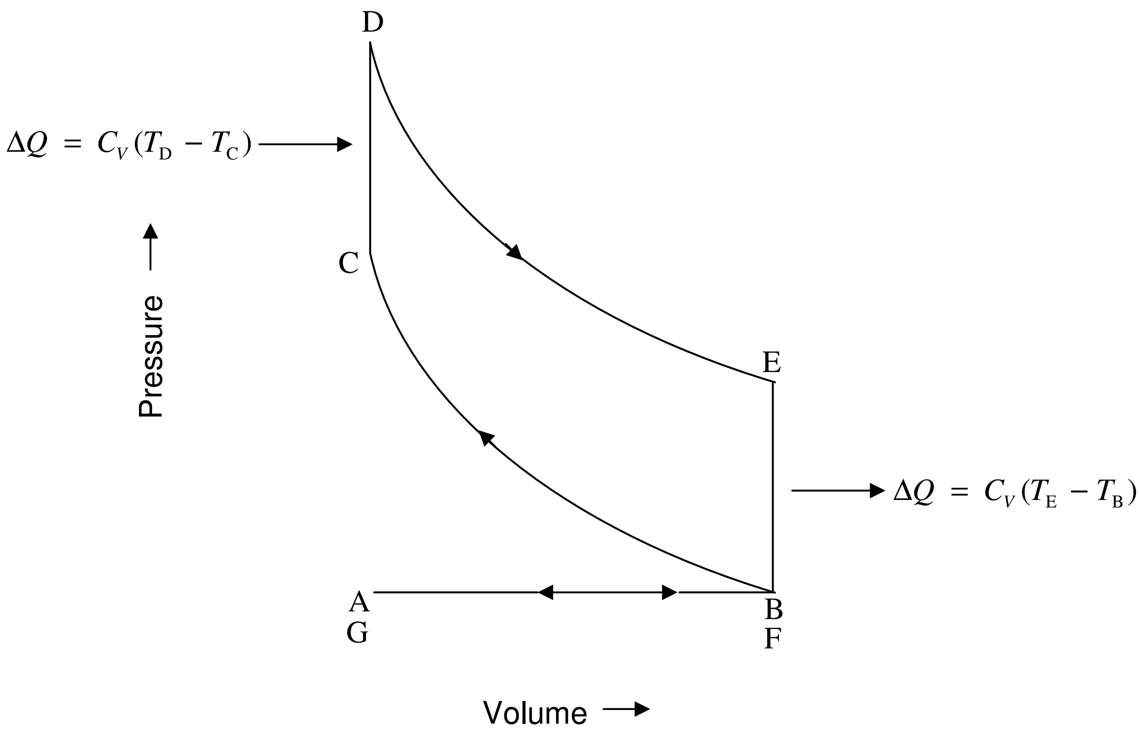


Figure 4

The cycle starts at A. From A to B the piston recedes and a valve is open, so that a misture of air and petrol (gasoline) is drawn in at constant (atmospheric) pressure. The temperature is typically somewhat above ambient temperature because of the previous operation of the cycle. At B, the valve is closes, and now from B to C a fixed mass of gas is compressed adiabatically, the temperature being a few hundred K. C is the point of maximum compression. At this point a spark is struck and the mixture is ignited. In effect heat is added to the system and the temperature goes up instantaneously to perhaps 2000 K at constant (small) volume. The gas, now having reached D, expands adiabatically to E, doing work, and the temperature drops somewhat. At E, a (second) valve opens, gas is expelled, the pressure drops to atmospheric, and the temperature drops to its original value. We are now at F. The piston pushes the remaining gas out, and we end at G. The cycle starts anew.

It is left as an exercise to show:

Net work done by the engine per cycle $= C_V(T_D - T_C)\left(1 - \frac{T_B}{T_C}\right).$

Volume of stroke $= V_B - V_C = V_B\left[1 - \left(\frac{T_B}{T_C}\right)^{1/(\gamma-1)}\right].$

Maximum pressure $= P_D = P_B \frac{T_D}{T_B}\left(\frac{T_C}{T_B}\right)^{1/(\gamma-1)}.$

Efficiency $= 1 - \left(\frac{V_C}{V_B}\right)^{\gamma-1} = 1 - \frac{T_B}{T_C}.$

In principle the efficiency could be very large if the temperature at C, at the end of the adiabatic compression, were high. In practice the temperature at the end of the adiabatic compression is limited (and therefore so is the efficiency) because, if the temperature were too high, the air-gasoline mixture would ignite spontaneously.

1.5 Diesel Cycle

This difficulty is avoided in the Diesel cycle in that, during the adiabatic compression stage to a high temperature, it is just air (not an air-fuel mixture) that is compressed. Only then, when the temperature is high, is fuel injected, which then immediately ignites. The cycle is shown in figure XI.6.

We start at A. A valve opens and the piston moves back, and pure air (no fuel) is sucked into the cylinder. This is followed by an adiabatic compression from B to C, which can reach a high temperature of 2000 K or so. At C a jet of liquid fuel is forced at high pressure into the cylinder by a pump that is operated by the engine itself. The fuel immediately ignites. The rate of injection is held so that the mixture expands at constant pressure until we reach D, at which point the injection of fuel is cut off and the gas expands adiabatically to E. A valve is then opened so that the pressure drops to atmospheric at F. The piston then pushes the remainder of the mixture out, and the cycle stars anew.

It is left as an exercise to show:

Net work done by the engine per cycle =

$$C_P\left[T_D - T_B\left(\frac{P_C}{P_B}\right)^{1-1/\gamma}\right] - C_V\left[T_D\left(\frac{P_B}{P_C}\frac{T_D}{T_B}\right)^{\gamma-1} - T_B\right].$$

Volume of stroke = $V_B - V_C = V_B\left[1 - \left(\frac{P_B}{P_C}\right)^{1/\gamma}\right].$

$$\text{Efficiency} = 1 - \frac{C_V\left[T_D\left(\frac{P_B}{P_C}\frac{T_D}{T_B}\right)^{\gamma-1} - T_B\right]}{C_P\left[T_D - T_B\left(\frac{P_C}{P_B}\right)^{1-1/\gamma}\right]}.$$

Have a look at
http://www.univ-lemans.fr/enseignements/physique/02/thermo/diesel.html

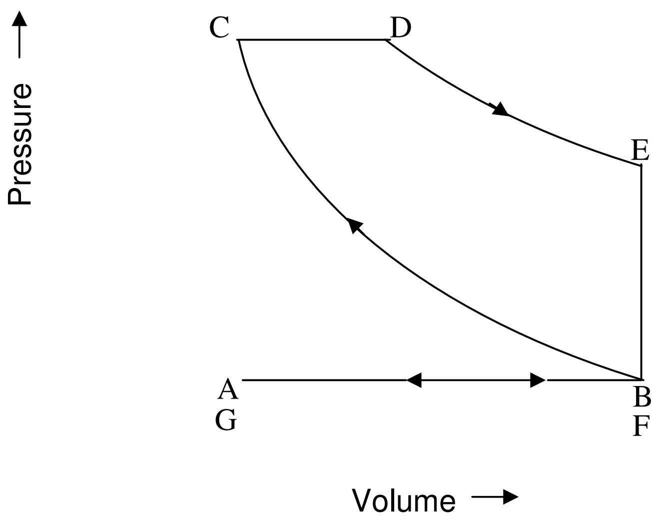


Figure 5

Exercise: Assuming $\gamma = 1.4$, what are the efficiencies of the Carnot, Otto and Diesel cycles running between 350 K and 2000 K? Assume for the Diesel cycle that the maximum pressure is 30 atmospheres. Assume for the Otto cycle that T_C =650 K.

1.6 Rankine Cycle

The *Titfield Thunderbolt* runs on an engine that slightly resembles the Rankine cycle.

The amount of work obtainable from an engine depends on the amount of the working substance and on the temperature. Internal combustion Otto and Diesel engines work at high temperatures, so they can be small. The steam engine is bulky but does not require high temperatures. The steam engine has a *boiler* (which, naturally, boils water into steam) and a *condenser* (which, naturally, condenses the steam back again to water).

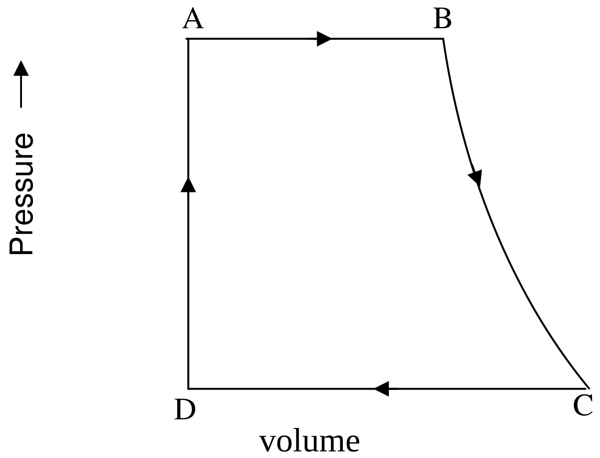


Figure 6

Steam from the boiler is drawn into a cylinder at constant pressure (A to B), at which point the intake valve is closed and the remaining expansion (B to C) is adiabatic, taking the temperature down to the temperature of the condenser. The section C to D corresponds to the condensation of the steam. From D to A the condensed water is transferred to the boiler, and the cycle starts again.

11.7 Useful Examples

It would probably *not* be a useful exercise to try to memorise the details of the several heat engine cycles described in this chapter. What probably *would* be a useful exercise is as follows. Note that in each cycle there are four stages, which, in principle at least (if not always in practice) are well defined and separated one from the next. These stages are described by one or another of an *isotherm*, an *adiabat*, an *isochor* or an *isobar*. It would probably be a good idea to ask oneself, for each stage in each engine, the values of ΔQ, ΔW and ΔU, noting, of course, that in each case, $\Delta U = \Delta Q + \Delta W$. In each case take care to note whether heat is added to or lost from the engine , whether the engine does work or whether work is done on it, and whether the internal energy increases or decreases. By doing this, one could then easily determine how much heat is supplied to the engine, and how much net work it does during the cycle, and hence determine the efficiency of the engine.

The following may serve as useful guidelines. In these guidelines it is assumed that any work done is reversible, and that (except for the steam engine or Rankine cycle) the working substance may be treated as if it were an ideal gas.

Along an *isotherm*, the *internal energy* of an ideal gas is unchanged. That is to say, $\Delta U = 0$. The *work* done (per mole of working substance) will be an expression of the form $RT \ln(V_2 / V_1)$, and the heat lost or gained will then be determined by $\Delta Q + \Delta W = 0$.

Along an *adiabat*, no heat is gained or lost, so that $\Delta Q = 0$. The expression for the work done per mole will be of the form $\frac{R(T_1 - T_2)}{\gamma - 1} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$, where V is the molar volume. Just be sure to understand whether work is done *on* or *by* the engine. The change in the internal energy (be sure to understand whether it is an increase or a decrease) is then given by $\Delta U = \Delta W$.

Along an *isochor*, no work is done. That is, $\Delta W = 0$. The heat lost or gained per mole will be of an expression of the form $C_V(T_2 - T_1)$, where C_V is the molar heat capacity at constant volume. The change in the internal energy (be sure to understand whether it is an increase or a decrease) is then given by $\Delta U = \Delta Q$.

Along an *isobar*, none of Q, W or U are unchanged. The work done per mole (*by* or *on* the engine?) will be an expression of the form $\Delta W = P(V_2 - V_1) = R(T_2 - T_1)$.

The heat added to or lost from the engine will be an expression of the form $C_P(T_2 - T_1)$, where C_P is the molar heat capacity at constant pressure. The change in the internal energy (be sure to understand whether it is an increase or a decrease) is then given by $\Delta U = \Delta Q + \Delta W$.

It might also be a good idea to try to draw each cycle in the $T : S$ plane (with the intensive variable T on the vertical axes). Indeed I particularly urge you to do this for the Carnot cycle, which will look particularly simple. Note that, while the *area* inside the cycle in the *P : V plane* is equal to the *net work* done on the engine during the cycle, the *area* inside the cycle in the *T : S plane* is equal to the *net heat* supplied to the engine during the cycle.

11.8 Heat Engines and Refrigerator

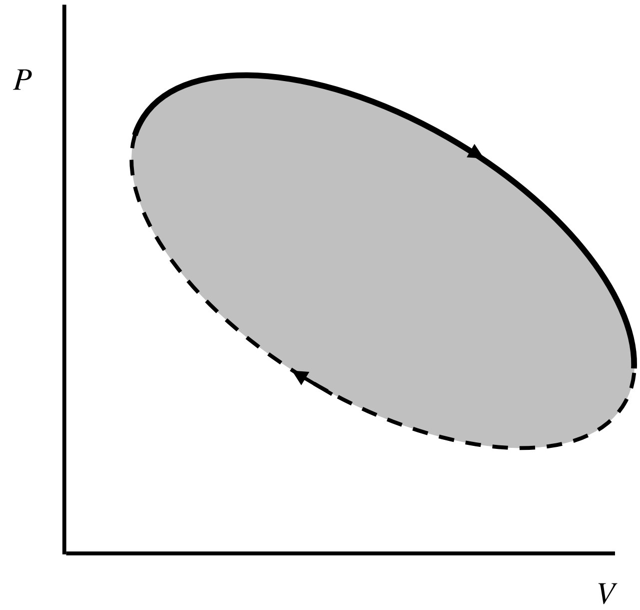


Figure 8

Figure 8 illustrates schematically the path taken by the state of a working substance is a generalized heat engine. In the upper part of the cycle (continuous curve) the working substance is expanding, and the machine is doing work. The work done **by** the engine is $\int P\,dV$, or the area under that part of the curve. In the lower part of the cycle (dashed curve) the working substance is being compressed; work is being done **on** it. This work is the area under the dashed portion of the cycle. The ***net*** work done **by** the engine during the cycle is the work done **by** the engine while it is expanding *minus* the work done **on** it during the compression part of the cycle, and this is the *area enclosed* by the cycle.

During one part of any heat engine cycle, heat is supplied **to** the engine, and during other parts, heat is lost **from** it. As described in Section 11.1, the *efficiency* η of a heat engine is defined by

$$\eta = \frac{\textbf{net}\ \textit{external work done}\ \textbf{by}\ \textit{the engine during a cycle}}{\textit{heat supplied}\ \textbf{to}\ \textit{the engine during a cycle}} .$$

Note that the word "net" does not appear in the denominator. The efficiency can also be *calculated* from

$$\eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}},$$

though I stress that this is not a *definition*.

In the *Carnot engine*, which is the most efficient conceivable engine for given source and sink temperature, the efficiency is

$$\eta = \frac{T_2 - T_1}{T_2},$$

where T_2 and T_1 are respectively the temperatures of the hot source and cold sink.

If the working substance is taken round a cycle in the *PV*-plane in the *counterclockwise* direction, the device is a *refrigerator*.

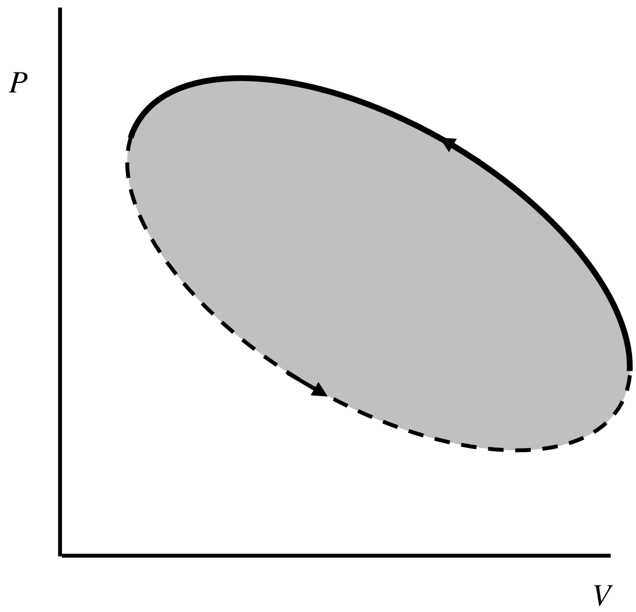


Figure 9

In that case the area enclosed by the cycle is equal to the net work that is done **on** the working substance. If the refrigerator operates on a reverse *Carnot* cycle, the working substance **takes in** (from whatever it is that it is trying to cool) a quantity of heat Q_1 as it expands isothermally from d to c (see figure , but with the arrows reversed) and **expels** a (greater) quantity of heat Q_2 as it is compressed isothermally from b to a. This quantity Q_2 is expelled into the room – which is why the room gets warmer when you switch on the fridge. (What – you never noticed?) The *refrigerating effect* is Q_1, since this is the quantity of heat taken in by the refrigerator from the body that is to be cooled.

The *coefficient of performance* of a refrigerator is defined by

$$\frac{\text{refrigerating effect}}{\text{net work done } \textbf{on} \text{ the engine during the cycle}}.$$

By the first law of thermodynamics, the denominator of the expression is $Q_2 - Q_1$, and for a reversible Carnot cycle, the entropy in equals the entropy out, so that $Q_2 / Q_1 = T_2 / T_1$. Therefore the coefficient of performance for a Carnot refrigeration cycle can be calculated from

$$\frac{T_1}{T_2 - T_1}.$$

This, of course, can be much greater than 1 – but no refrigerator working between the same source and sink temperatures can have a coefficient of performance greater that that of a reversible Carnot refrigerator.

Of course the working substance in a real refrigerator ("fridge") is not an ideal gas, nor does one follow a Carnot cycle – there are too many practical difficulties in the way of achieving this ideal dream. As mentioned elsewhere in this course, I am not a practical man and I am not suited to describing real, practical machines. The fundamental principles described in this section do, of course, still apply in the real world! In a real refrigerator, the working substance (the *refrigerant*) is a volatile fluid which is vaporized in one part of the operation and condensed to a liquid in another part. In industrial refrigerators, the refrigerant may be ammonia, but this is considered to be too dangerous for domestic use. "Freon", which was a mixture of chlorofluorocarbons, such as CCl_2F_2, was in fashion for a while, but escaping chlorofluorocarbons have been known for some time to cause breakdown of ozone (O_3) in the atmosphere, thus destroying our protection against ultraviolet radiation from the Sun. The chlorofluorocarbons have been largely replaced by hydrofluorocarbons, such as $C_2H_2F_4$, which are believed to be less damaging to the ozone layer. The exact formula or mixture is doubtless a trade secret.

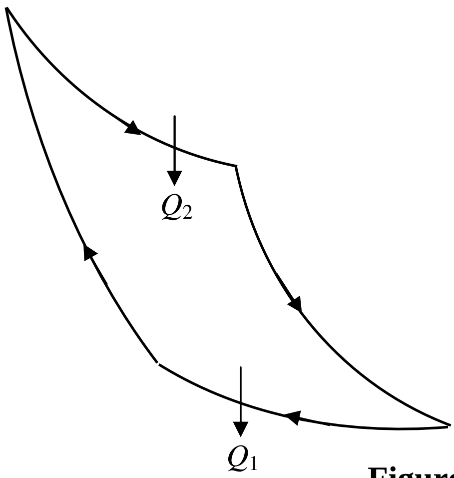
The fluid is forced around a system of tubes by a pump called the *compressor*. Shortly before the fluid reaches the freezer it is in liquid form, moving along some rather narrow pipes. It is then forced through a nozzle into a system of wider pipes (the *evaporator*) surrounding the freezer, and there it vaporizes, taking heat from the food and from the air in the freezer. A fan may also distribute the cooled air throughout the rest of the refrigerator. After leaving the freezer, the vapour returns to the *compressor*, where it is, of course, compressed (which is why the pump is called the compressor). This produces heat, which is dissipated into the room as the fluid is forced through a series of pipes and vanes, known as the *condenser*, at the rear of the fridge, where the fluid condenses into liquid form again. The cycle then starts anew.

The following summary of Carnot heat engines and refrigerators may be helpful. (But just remember that, while Carnot cycles are the most efficient engines and refrigerators for given source and sink temperatures, the practical realization of a real engine or refrigerator may not be identical to this theoretical ideal.)

Notation: T_2 = hotter temperature
T_1 = cooler temperature
Q_2 = heat gained or lost at T_2
Q_1 = heat gained or lost at T_1

$\Delta S = 0$ $$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}.$$

11.9 Heat Engine:



$Q_{\text{in}} = Q_2$

$Q_{\text{in}} > Q_{\text{out}}$

$Q_{\text{out}} = Q_1$

Figure 10

$\Delta U = 0$ Net work done **by** engine $= Q_2 - Q_1$.

$$\text{Efficiency } \eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = \frac{Q_2 - Q_1}{Q_2} = \frac{T_2 - T_1}{T_2}.$$

11.10 Refrigerator

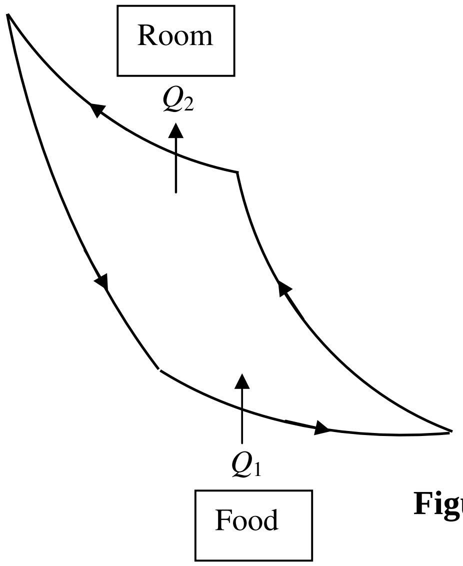


Figure 11

$Q_{in} = Q_1$

$Q_{out} > Q_{in}$

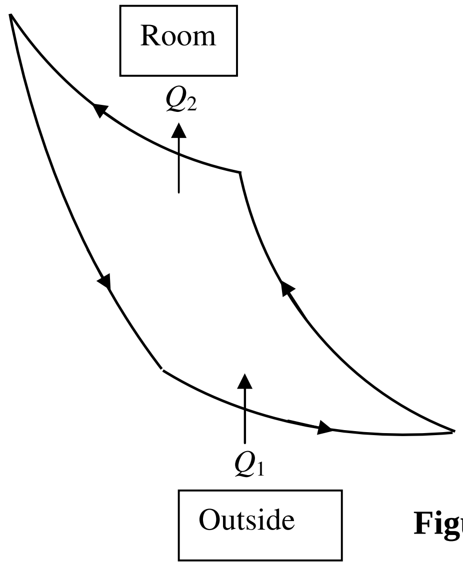
$Q_{out} = Q_2$

$\Delta U = 0$ Net work done **on** refrigerator $= Q_2 - Q_1$.

Coefficient of Performance $P = \dfrac{Q_{in}}{Q_{out} - Q_{in}} = \dfrac{Q_1}{Q_2 - Q_1} = \dfrac{T_1}{T_2 - T_1}$.

Heat Pump:

The principle of a heat pump is the same as that of a refrigerator, except that its purpose is different. The purpose of a refrigerator is to extract heat from something (e.g. food) and so to make it colder. That the heat so extracted goes into the room to make the room warmer (at least in principle) is incidental. The important thing is how much heat is extracted from the food, and that is why it is appropriate to define the coefficient of performance of a refrigerator as the *refrigerating effect* (i.e. Q_1) divided by the net work done on the refrigerator, per cycle. But with a heat pump, the object is the *heat the room* by extracting heat from outside. That the outside may become cooler (at least in principle) is incidental. Thus, for a heat pump, the appropriate definition of the coefficient of performance is the *heating effect* (i.e. Q_2) divided by the net work done on the refrigerator, per cycle.



$$Q_{\text{in}} = Q_1 \qquad Q_{\text{out}} > Q_{\text{in}}$$

$$Q_{\text{out}} = Q_2$$

Figure 12

$\Delta U = 0$ Net work done **on** heat pump $= Q_2 - Q_1$.

$$\text{Coefficient of Performance } P = \frac{Q_{\text{out}}}{Q_{\text{out}} - Q_{\text{in}}} = \frac{Q_2}{Q_2 - Q_1} = \frac{T_2}{T_2 - T_1}.$$

You can see from this equation that, the warmer it is outside (T_1), the greater the coefficient of performance. You may therefore wonder if it is practical to use a heat pump to heat a building in a cold climate, such as the Quebec winter. And, if it isn't, can one devise an engine that is simultaneously a refrigerator and a heat pump; that is to say, it extracts heats from (i.e. cools) the food, and transfers this heat (plus a little bit more because of the work that is done on the refrigerator/heat pump) into the room in order to heat the room effectively. There's an answer to that in an article in the Victoria *Times-Colonist* of June 11, 2006, which I reproduce, with permission, below.